

March 1998

IFP-757-UNC

# Kaon Spontaneous CP Violation Reevaluated

Paul H. Frampton and Masayasu Harada

*University of North Carolina, Chapel Hill, NC 27599-3255*

## Abstract

The CP parameters  $\epsilon$  and  $\frac{\epsilon'}{\epsilon}$  are calculated in the aspon model of spontaneous CP violation, a model which solves the strong CP problem. A new range for the scale of spontaneous breaking of CP is found. It is shown that  $\frac{\epsilon'}{\epsilon}$  is suppressed by  $\sim x^2 v^2 / (\kappa^2 \sin^5 \theta_C) \sim 5 \times 10^{-3}$  relative to the Standard Model. If experiment finds that  $\frac{\epsilon'}{\epsilon}$  is  $10^{-4}$  or greater in magnitude, it will mean that the present approach to spontaneous CP violation is excluded.

Typeset using REVTeX

The origin of CP violation is still not well understood. It could arise from explicit breaking, for example in the KM mechanism [1] of quark flavor mixing of three families. Alternatively, it may arise from spontaneous CP breaking, for example as in the aspon model [2]. This model solves the strong CP problem and provides a mechanism for weak CP violation which can explain the parameter  $\epsilon_K$  in the kaon system [3]. It also predicts very small CP asymmetries in  $B^0 - \bar{B}^0$  decays [4] and production of exotic particles at the LHC [5].

The purpose of this article is to reevaluate the predictions of the aspon model for the  $K^0$  system, particularly  $\epsilon$  and  $\text{Re}(\frac{\epsilon'}{\epsilon})$  more carefully than has been done before. This will lead to some new predictions and constraints on the parameters of the model.

The aspon model possesses a gauge symmetry  $(SU(3)_c \times SU(2)_L \times U(1)_Y) \times U(1)_X$ . All the particles of the Standard Model with three families, including one doublet Higgs scalar, have aspon charge  $X = 0$ . The additional states are a non-chiral doublet  $Q = (U, D)$  of heavy quarks with  $X = 1$  and two complex singlet Higgs scalars  $\chi^\alpha (\alpha = 1, 2)$  with  $X = 1$ . With this arrangement, the strong CP problem is solved (given a certain constraint on the parameters) because at leading-order the quark mass matrix has a real determinant. The  $\chi^\alpha$  develop complex VEVs  $\langle \chi^\alpha \rangle = \rho^\alpha e^{i\phi^\alpha}$  with  $(\phi^1 - \phi^2) \neq 0$ , thus breaking CP and giving the  $U(1)_X$  gauge boson -the ‘‘aspon’’- a mass through the Higgs mechanism.

Let the Yukawa couplings of the light to heavy quarks be written  $h_i^\alpha q_L^i Q_R \chi^\alpha$  and define  $x_i = \Sigma_\alpha h_i^\alpha \langle \chi^\alpha \rangle / M$  where  $M$  is the mass of  $Q$  then the requirements of strong CP and naturalness constrain  $|x_i|^2$  to be [6]

$$3 \times 10^{-5} < |x_i|^2 < 10^{-3} \quad (1)$$

for each  $i = 1, 2, 3$  (hereafter we suppress the subscript and modulus sign on  $|x_i|^2 \rightarrow x^2$ ).

Let us first evaluate  $\epsilon_K$  given by:

$$|\epsilon_K| = \frac{1}{\sqrt{2}\Delta m_K} \left( \text{Im} M_{12} + 2 \left( \frac{\text{Im} A_0}{\text{Re} A_0} \right) \text{Re} M_{12} \right) \quad (2)$$

The second term is an order of magnitude smaller than the first because  $(\text{Im} A_0)/(\text{Re} A_0)$

is orders of magnitude less than  $|\epsilon_K|$  and  $ReM_{12} \sim \Delta m_K$ . The first term then gives, with  $\Delta m_K = 3.5 \times 10^{-15} \text{GeV}$  and the tree-level aspon exchange (Fig. 1.)

$$|\epsilon_K| = \frac{1}{\sqrt{2}\Delta m_K} Im(x_1^* x_2)^2 \frac{f_K^2}{3} m_K \frac{2}{\kappa^2} \quad (3)$$

with  $\kappa = \sqrt{(\rho_1^2 + \rho_2^2)}$ . Writing  $Im(x_1^* x_2)^2 \rightarrow x^4$  and using  $|\epsilon_K| = 2.26 \times 10^{-3}$  gives the relationship

$$\kappa/x^2 = 2.9 \times 10^7 \text{GeV}. \quad (4)$$

Thus the symmetry breaking scale  $\kappa$ , given the range of  $x^2$  in Eq. (1) satisfies [7]

$$29 \text{TeV} > \kappa > 870 \text{GeV}. \quad (5)$$

The aspon mass  $M_A = g_A \kappa$  may be estimated, taking *e.g.*  $g_A = 0.3 (= e)$  as  $8.7 \text{TeV} > M_A > 260 \text{GeV}$ .

To evaluate  $Re(\frac{\epsilon'}{\epsilon})$  requires the study of several Feynman diagrams [8], and their comparison to the Standard Model. Recall that the most recent evaluations in runs at CERN (NA31) [9] and FNAL (E731) [10] give the results  $Re(\frac{\epsilon'}{\epsilon}) = (23 \pm 3.6 \pm 5.4) \times 10^{-4}$  and  $Re(\frac{\epsilon'}{\epsilon}) = (7.4 \pm 5.2 \pm 2.9) \times 10^{-4}$  respectively, where the first error is statistical and the second is systematic. These results are consistent within two standard deviations; the error is expected to be reduced to  $1 \times 10^{-4}$  in foreseeable future experiments.

We first consider the two tree diagrams shown in Fig. 2. An estimate of Fig. 2(a) is  $(x^4 v^2 / \kappa^2) \leq 7 \times 10^{-11}$ , where we use Eq. (4), to be compared with  $\frac{\alpha_s}{4\pi} (\sin \theta_C)^5 \simeq 10^{-5}$  for the largest (gluon penguin) Standard Model contribution. The tree diagram of Fig. 2(b) can be made real by phase rotations of the quark fields.

Also contributing to  $Re(\frac{\epsilon'}{\epsilon})$  at one loop level are the penguin diagrams of Fig. 3, and the box diagrams of Fig. 4.

Beginning with the penguins in the Standard Model, the gluon penguin (Fig. 3(a)) is dominated by charm because  $Im(V_{ud}^* V_{us}) = 0$  and because  $Im(V_{cd}^* V_{cs}) \simeq Im(V_{td}^* V_{ts}) \sim \sin^5 \theta_C$  while the Feynman amplitude is an order of magnitude larger for  $m_c$  than  $m_t$ . On

the other hand, the electroweak Z-penguin (Fig. 3(b)) is proportional to  $m_q^2$  and is dominated by top; again it is  $\sim \sin^5 \theta_C$  and tends to cancel the gluon penguin [11]. Of course, Figs. 3(c) and 3(d) do not exist in the Standard Model.

In the aspon model, the imaginary parts of the penguin diagrams arise quite differently from in the Standard Model, because the CKM matrix elements are replaced by new expressions at the vertices, for example:

$$\text{Im}(V_{us}^* V_{ud}) = 0 \quad (6)$$

$$\text{Im}(V_{cs}^* V_{cd}) \sim -A^2 \rho(1 - \rho)x^2 \sin^6 \theta_C \quad (7)$$

$$\text{Im}(V_{ts}^* V_{td}) \sim +A^2 \rho(1 - \rho)x^2 \sin^6 \theta_C \quad (8)$$

$$\text{Im}(V_{Us}^* V_{Ud}) = O(x^4) \quad (9)$$

As a consequence, the gluon penguin (Fig. 3(a)) is again dominated by charm while the Z-penguin (Fig. 3(b)) is dominated by top. But the replacement of the usual CKM matrix elements means a suppression relative to the Standard Model by a factor  $x^2 \sin \theta_C \leq 2 \times 10^{-4}$ . We expect a partial cancellation between the gluon and Z penguins similar to that in the Standard Model.

For the penguin diagrams peculiar to the aspon model (Figs. 3(c) and 3(d)), we can dismiss the diagram of Fig. 3(d) as negligible, being of order  $x^4$ . On the other hand, the new gluon penguin of Fig. 3(c) gives a contribution of order  $x^2 v^2 / \kappa^2$  so that its suppression relative to the Standard Model gluon penguin is parametrized by  $x^2 v^2 / (\kappa^2 \sin^5 \theta_C) \leq 5 \times 10^{-3}$ . We find that this diagram (Fig. 3(c)) is therefore the largest contributor to  $\text{Re}(\frac{\epsilon'}{\epsilon})$  in the aspon model. At the same time, we see that  $\text{Re}(\frac{\epsilon'}{\epsilon})$  is highly suppressed, with a magnitude  $\leq 10^{-5}$ .

Finally, there are the box diagrams of Fig. 4, where Fig. 4(c) is peculiar to the aspon model. This last figure, Fig. 4(c), is actually proportional to  $x^4$  and hence negligible.

In the Standard Model the box diagrams Figs. 4(a) and 4(b) are smaller than the penguin amplitudes of Figs. 3(a) and 3(b) in their contribution to  $Re(\frac{\epsilon'}{\epsilon})$  because of the interplay between the CKM elements and the masses  $m_q$  interior to the diagram. The Feynman amplitude  $\sim (m_q/m_W)^2$  for the same quark on each side and  $\sim (m_{q_1}/m_W)^2 \ln(m_{q_2}/m_{q_1})$  for  $m_{q_2} > m_{q_1}$  with different quarks on the two sides. In the aspon model, it is straightforward to see that for similar reasons the box diagrams do not compete with the penguins.

In summary, we have found that the symmetry-breaking scale  $\kappa$  for spontaneous CP violation should satisfy  $29TeV > \kappa > 870GeV$ . It has also been concluded that  $|Re(\frac{\epsilon'}{\epsilon})| \leq 10^{-5}$  in this model. While  $Re(\frac{\epsilon'}{\epsilon})$  is not expected to vanish identically, it does correspond closely to the superweak model prediction [12].

Discovery experimentally of  $|Re(\frac{\epsilon'}{\epsilon})| > 10^{-4}$  would certainly mean that this type of approach is ruled out.

This work was supported in part by the US Department of Energy under Grant No. DE-FG05-85ER-40219.

## REFERENCES

- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [2] P.H. Frampton and T.W. Kephart, Phys. Rev. Lett. **66**, 1666 (1991).
- [3] P.H. Frampton and D. Ng, Phys. Rev. D **43**, 3034 (1991).
- [4] A.W. Ackley, P.H. Frampton, B. Kayser and C.N. Leung, Phys. Rev. D **50**, 3560 (1994).
- [5] P.H. Frampton, D. Ng, T.W. Kephart and T.J. Weiler, Phys. Rev. Lett. **68**, 2129 (1992).
- [6] P.H. Frampton and S.L. Glashow, Phys. Rev. D **55**, 1691 (1997).
- [7] We have corrected an error in the evaluation of  $Im\lambda_{u,t,U}$  following Eq.(33) in [3]
- [8] A.J. Buras, M. Jamin and M.E. Lautenbacher, Phys. Lett. B **389**, 749 (1996); M. Ciuchini, Nucl. Phys. B, Proc. Suppl. **59**, 149 (1997); M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, Z. Phys. **C68**, 239 (1995); S. Bertolini, M. Fabbrichesi and J.O. Eeg, hep-ph/9802405; S. Bertolini, J.O. Eeg, M. Fabbrichesi and E.I. Lashin, Nucl. Phys. **B514**, 63, 93 (1998).
- [9] G.D. Barr *et al.*, Phys. Lett. B **317**, 233 (1993).
- [10] L.K. Gibbons *et al.*, Phys. Rev. Lett. **70**, 1203 (1993); L.K. Gibbons *et al.*, Phys. Rev. **D55**, 6625 (1997).
- [11] J.M. Flynn and L. Randall, Phys. Lett. B **224**, 221 (1989); erratum **235**, 412 (1990): G. Buchalla, A.J. Buras and M.K. Harlander, Nucl. Phys. B **337**, 313 (1990).
- [12] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).

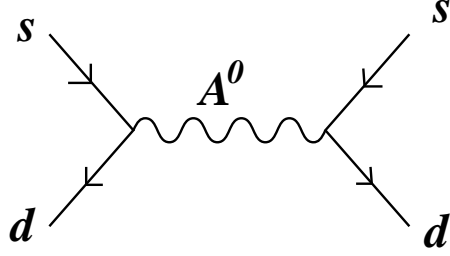


FIG. 1. Tree level aspon ( $A^0$ ) exchange contribution to  $K^0-\bar{K}^0$  mixing.

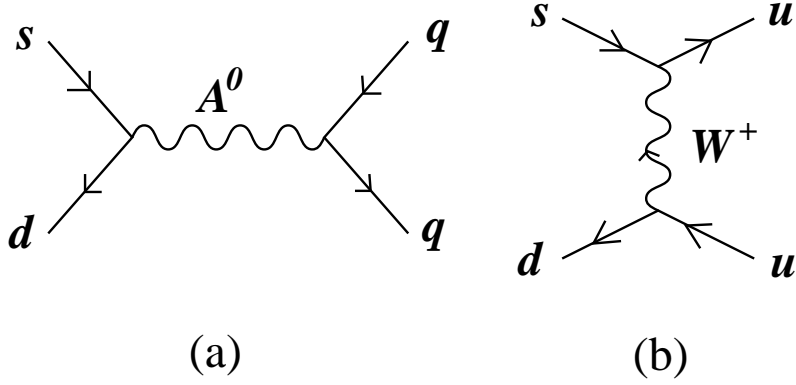


FIG. 2. Tree level contributions to  $\epsilon'/\epsilon$ : (a) aspon exchange and (b)  $W$  exchange.

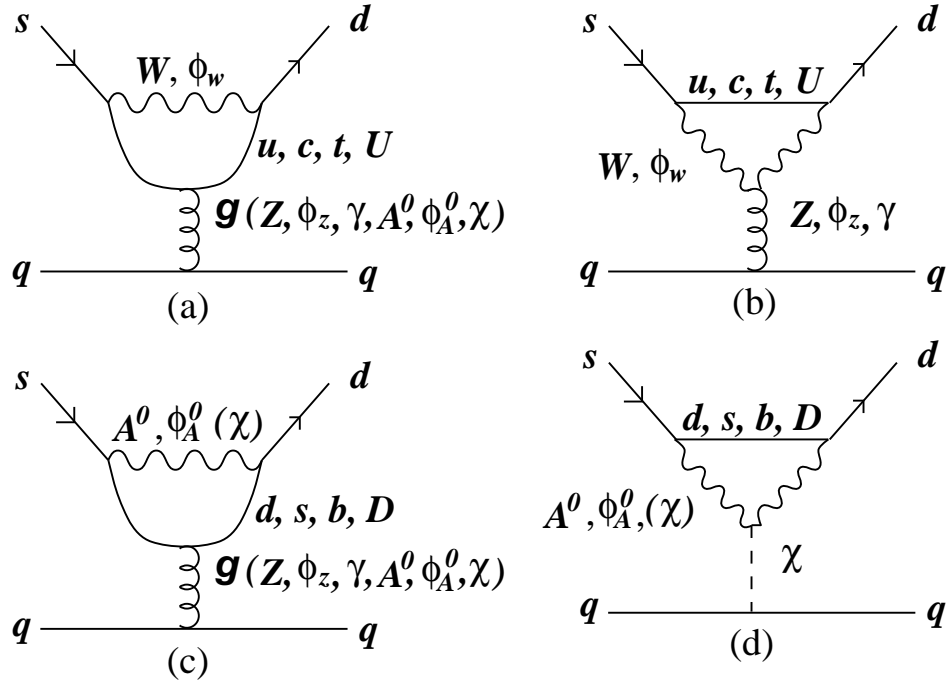


FIG. 3. Penguin diagram contributions to  $\epsilon'/\epsilon$ .  $\phi_A^0$ ,  $\phi_W$  and  $\phi_Z$  are the would-be Nambu-Goldstone bosons absorbed into the aspon ( $A^0$ ),  $W$  and  $Z$ , respectively.  $\chi$  denotes the massive scalar bosons with aspon charge. scalar bosons.



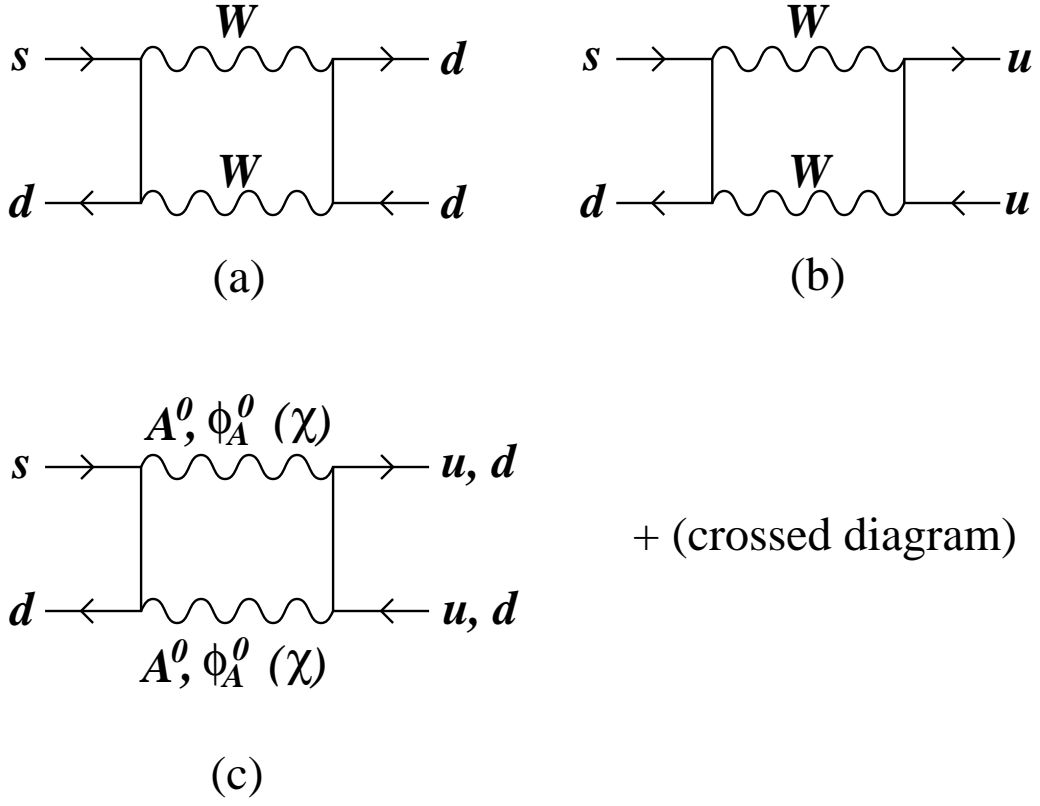


FIG. 4. Box diagram contributions to  $\epsilon'/\epsilon$ .